Teaching ‘work’ as the measure of work

NIKOS KANDERAKIS¹, ELINI DANILI²

¹University of Western Macedonia (external collaborator) Greece
nikanderakis@yahoo.gr

²Headmistress of the Experimental School of Athens Greece
eleni.danili@gmail.com

ABSTRACT
This article describes a teaching approach to the concept of ‘work’ for the upper secondary Greek school students. The teaching material presented aims at (a) teaching ‘work’ as a measure of work, and (b) clarifying when (under what conditions) ‘work’ is produced. The design of the teaching activities has been inspired by the history of mechanics and guided by the studies and the arguments of the 19th century’s engineers who created the magnitude of ‘work’ in an attempt to measure the quantity of work produced by motor engines, humans and animals. The activities include inquiries and experiments (or rather experiences), where students estimate quantitatively the work done by various acting agents in different circumstances. Students first study the cases of the vertical elevation of various weights by a human and then explore cases of other kinds of movements (e.g. a horizontal displacement of a weight) or other acting agents (e.g. motor engines). At the end, there is a metacognition discussion.

KEY WORDS
Mathematics school knowledge, pedagogic practices, preschool education, social context

RÉSUMÉ
Cet article décrit une méthode d’enseignement du concept de «travail» aux élèves grecs de l’enseignement secondaire (niveau lycée). Les deux principaux objectifs
de cette approche sont les suivants: (a) d’introduire le concept «travail» comme une mesure de travail, et (b) les élèves à préciser les conditions que doit remplir une force afin de produire «travail». La conception de l’approche pédagogique a été inspirée par l’histoire de la mécanique et guidé par les études et les arguments des ingénieurs du 19eme siècle, qui avaient créé le grandeur du «travail» dans une tentative de mesurer le travail. Il contient une série d’activités d’investigation et d’expériences, pendant lesquelles les élèves doivent estimer quantitativement le travail accompli. Les élèves d’abord étudient le cas de l’élévation verticale de poids différents par les humains et ensuite exploitent d’autres types de mouvements (p.e., un déplacement horizontal d’un poids) ou d’autres agents actives (p.e., machines à moteur). À la fin de l’enseignement, il y avait une discussion métacognitive.

Mots-Clés
Travail, énergie, énergie cinétique, force vive, vis viva

Introduction
In the first half of the 19th century, French engineers created the physical magnitude of ‘work’ in order to measure the work produced by motor engines, men, and animals. This initial function of ‘work’ is usually missing from the physics school textbooks. ‘Work’ is mainly introduced as a purely theoretical magnitude related to energy transfer, without empirical meaning (Baltas, 1988), that is without direct connection to the physical world. In our teaching approach, which is addressed to students of the upper secondary Greek school (ages 15-16), the physical magnitude of ‘work’ is constructed using specifically this function of ‘work’, that is to measure work (labor). The proposed teaching sequence uses the student’s everyday experience, and is guided by the studies and the arguments of the engineers who had created the physical magnitude of ‘work’. The article is composed of three main parts: a brief historical review of the creation of ‘work’ as a physical magnitude, a brief survey of the proposals to use history of science material in the science classroom, and finally the methodology and activities of the proposed teaching approach.

A Short History of the Creation of ‘Work’
‘Work’ as a magnitude of mechanics was created at the beginning of the 19th century, when two different 18th century practices were met to build a general theory for the moving engines. Until then, the only available theory was statics. But in statics, machines were studied in a stationary and equilibrium state, and not in motion.
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by philosophers and mathematicians, and concerned calculations on a moving body’s ‘force’. The second one, which was empirical, was set out by engineers, and it calculated work, as well as effectiveness, of motor engines (water-wheels and steam engines).

Calculating vis-viva

For Descartes and Cartesians, the measure of a moving body’s ‘force’ (which determined the outcome of its collisions) was the body’s ‘quantity of motion’ (\(m \cdot v\)). Leibniz and Leibnizians, on the other hand, distinguished two kinds of ‘force’; ‘dead force’, which was the force of statics, and ‘living force’, which was the ‘force’ of the moving bodies. ‘Living force’ could be measured either by its cause or by the effect it produced. In the case of a body’s fall, the cause of the acquired ‘living force’ was measured by the product of the body’s weight multiplied by the height of its fall. In the case of a body thrown straight up, the effect produced by the body’s ‘living force’ was measured again by the product of the body’s weight and the height of its elevation. In both cases ‘living force’ turned out to be proportional to the body’s mass and the square of its velocity, that is \(m \cdot v^2\) (Leibniz, 1989a, 1989b; Kanderakis 2008). In general, ‘living force’ was measured by the product of the Newtonian force which moved the body against impediments and the body’s displacement (\(F \cdot s\) or the integral \(\int F \cdot s\)), (Bernoulli, 1727). According to Leibniz, the total ‘living force’ in the Universe remained constant. Even in non-elastic collisions, ‘living force’ did not perish, but was absorbed by the bodies’ tiny parts and became invisible (Leibniz, 1916).

There was strong opposition to Leibniz’s views, not only from the Cartesians, but also from the Newtonians. The former insisted on the ‘quantity of motion’ as the measure of ‘force’, whereas the latter claimed that ‘force’ could not be conserved at all. The striking feature of this controversy was the great confusion regarding the meaning of the word ‘force’. For Cartesians the word signified ‘quantity of motion’ (\(m \cdot v\), usually without direction), for Leibnizians ‘living force’ (\(m \cdot v^2\)), and for Newtonians sometimes ‘momentum’ (\(m \cdot v\), with direction) and sometimes common Newtonian force (Reid, 1748). In the course of the century, the theory of ‘living forces’ became obsolete, and was only employed in special cases such as elastic collisions (Ilfracombe, 1970). The situation did not significantly change, even when Poleni and Gravesande, letting hard balls fall on soft substances, showed experimentally that the effects of collisions were proportional to the balls’ weight and height, and consequently to their ‘living forces’ (Kanderakis, 2014).

Measuring the effectiveness of motor engines

The first systematic study of motor-engines was carried out by the French engineer Antoine Parent in 1704, and concerned water-wheels. Parent focused on the impact force the moving water exerted, and ignored the action of the water’s weight (in the
case of an overshot wheel). He also calculated the effect of the wheel with a peculiar ‘quantity of motion’ (the product of the elevating weight and its velocity), which was related to the instantaneous action of the impressed forces and did not measure the wheel’s accumulated outcome at a given time interval (Parent, 1704). This was the predominant way to analyze water-wheels for the whole 18th century (Reynolds, 1973).

The engineers Jean Charles Borda in France and John Smeaton in England managed to calculate this accumulated outcome of the wheel work using the ‘living forces’ but their studies were ignored by natural philosophers for many decades (Reynolds, 1973).

The second important kind of motor engines of the era were steam-engines. Initially, they were constructed in Britain to drain deep mines, and their effectiveness (‘duty’) was measured by the outcome they produced: the raising of water. Specifically, their ‘duty’ was measured by the product of the weight raised and the height of its elevation against either a given time interval or burning a certain amount of fuel. For example, a certain steam-engine raised 30 pounds of water one foot high in a minute (its ‘duty’ was 30 foot-pounds per minute). Another steam-engine raised 50 pounds of water one foot high burning a bushel of coal (its ‘duty’ was 50 foot-pounds per bushel of coal). (Dickinson, 1963; Hills 1970; Hills & Pacey, 1972).

In the second half of the 18th century, due to the appearance of the industrial revolution in Britain, steam engines were employed to move a variety of machinery, such as the different machines in a cotton mill or in an ironwork factory. Engineers, however, retained the old measure from the mining industry to estimate their steam-engines’ new outcomes. Consequently, the elevation of a weight formed the model work for all work done by steam engines, and its measure (weight multiplied by height) was used to calculate all kinds of work produced by these engines. The horse (or horsepower), another measure of the engines’ effectiveness, probably played an important role to this extension of use. Steam engines replaced horse-driven engines; therefore, the horse was a usual measure of the engines’ potential. Although there were initially diverse views on how many foot-pounds per minute a horse produced, in the end, Boulton and Watt’s value (33000 foot-pounds per minute) prevailed (Hills & Pacey, 1972). Yet, this measure was only a practical tool, strictly located within the realm of steam engines and it could not be generally applied to the work done by any acting agent, or at least by any motor engine. It is noteworthy, that when some engineers, who had good knowledge of the new science, tried to analyze the steam engine’s function, they used statics and the equilibrium of forces, all of which were rather inadequate to meet the needs of the engineers in the mines or the industries (see for example Desaguliers, 1734 and Triewald, 1734).

**Creating a general theory for the engines in motion**

The general theory for the engines in motion, and ‘work’ as a magnitude of mechanics,
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were created in France, within the realm of the French higher technical education. The main characters were Louis Navier, Gaspard Coriolis, and Jean Victor Poncelet, engineers and teachers at higher engineering schools. Their aim was to create a theory for the motor engines (or the moving engines in general), directly available for practical applications, or, as Poncelet put it, an ‘industrial mechanics’ (Chatzis, 1997; Darrigol, 2001).

Navier edited, in 1819, a new publication of Bellidor’s ‘Architecture Hydraulique’ (an old and popular engineering textbook), and framed a general theory for the moving engines in the footnotes. His main magnitudes were the ‘living force’ and the ‘quantity of action’. The latter was equal to the force applied by the engine multiplied by the displacement it produced, and it measured the engine’s work. Until then, as Navier wrote, the work (labor) of a motor engine was estimated by its outcome, whatever that outcome might be. However, using the ‘quantity of action engineers would have a common measure for the motor engines’ work suitable for any kind of engine and for any use. According to Navier, the work of any engine would be to overcome a resistance for a certain space, and it was equivalent to the elevation of a weight (equal to the resistance) to a certain height (equal to the space traveled) not only in theory but also in practice (employing pulleys and ropes). As a result, the elevation of a weight was the model work for any other work (Bellido, 1819; Kanderakis, 2014).

Navier connected ‘quantity of action’ to the changes of the ‘living force’. If a force F was acting on an unimpeded body with mass m, the body, traveling a distance p, would get a velocity U, and his ‘living force would be \( mU^2 = 2Pp \). Thus, the produced ‘living force’ would be proportional to the ‘quantity of action’ spent (Bellido, 1819).

Coriolis published in 1829 a book entitled ‘Du Calcul de l’Effet des Machines’ to develop a theory for motor engines. His main magnitude was named ‘work’ (travail) in order to hold something from the everyday sense of work, and was equal to \( \int Pds \), where \( P \) was the acting force and \( ds \) the infinitesimal displacement in its direction. ‘Work’ would measure the work of any acting agent: motor engine, man, or animal. According to Coriolis, ‘work’ (and work) had to be connected with an acting force and a distance travelled, happening at the same time. We could not have work and could not calculate ‘work’ in the case of a force acting on an unmoved body, or in a displacement of a body without resistance (and moving force). In order to have ‘work’, both force and displacement had to exist (Coriolis, 1829).

Another innovation of Coriolis was the change of the ‘living force’ from \( \frac{pv^2}{g} \) to \( \frac{pv^2}{2g} \), where \( p \) was the body’s weight and \( \frac{p}{g} \) its mass (in modern symbolism from \( mv^2 \) to \( \frac{1}{2} mv^2 \)). This small change, he wrote, introduced great simplification to the expression of the principles of mechanics. For example, the ‘equation of the living forces’

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2 Coriolis had been working on these subjects since 1819, and his manuscripts were circulating for years among his colleagues (Grattan-Guinness, 1984).
\[ \sum F ds - \sum F' ds' = \sum pv^2/2g - \sum pv_0^2/2g, \]

where \( P \) were the moving forces and \( P' \) the resisting forces, could be expressed as follows: in a system of moving bodies, the difference between the ‘work’ of the moving forces and the ‘work’ of the resisting forces was equal to the change of the ‘living forces’ of all the system’s bodies (Coriolis, 1829).

Poncelet published in 1829 a lithographed book on the engines in motion, which was printed in 1870 under the title ‘Introduction à la Mécanique Industrielle Physique ou Expérimentale’. The subject of the book, according to Poncelet, was industrial mechanics, that is the science of ‘work’. ‘Work’ was defined as the product of the acting force multiplied by the displacement in the force’s direction, and it presupposed both a resistance to be overcome and a space to be travelled. A steady movement without resistance, due to the inertia of matter, would not need an effort, and would not require work. On the other hand, if a man applied an effort (force) or kept a weight without movement, he would not really work because the acting agent could be replaced with a stationary support (rope, pillar etc), which, of course, would not work (Poncelet, 1870; Kanderakis, 2014). This quantity, according to Poncelet, could be used as the measure of the work carried out by any kind of motor engine, and it could also measure the work of men and animals. This measure corresponded to what it was paid in money when forces were employed (Poncelet, 1870).

Both Coriolis and Poncelet criticized Coulomb who, in the case of a man who was moving a body horizontally, had calculated the work done by the product of the body’s weight multiplied by its displacement. According to them, it was the moving force and not the moving body’s weight, which had to be multiplied by the displacement. As Coriolis remarked, a horse, doing the same work, could drag horizontally very different weights (from a carriage to a boat), depending on the circumstances, i.e. friction. Consequently, what mattered was the resistance confronted (or the moving force) and not the body’s weight (Coriolis, 1829; Poncelet, 1870; Kanderakis, 2014).

Eventually, the lectures of Navier, Coriolis, and Poncelet, and also the lectures of their colleagues who adopted their ideas, but mainly their textbooks, spread the new concepts within the realm of engineering, and finally within the communities of rational mechanics (Darrigol, 2001).

Summarizing, the creation of ‘work’ as a magnitude incorporated in the theory of mechanics, can be understood as a procedure of four stages:

i. Initially, the engineers in the mines created the magnitude ‘weight multiplied by height’ to measure the work produced by steam engines which were elevating weights.

ii. This magnitude was extended by the same engineers to measure any work performed by steam engines.

iii. This same magnitude was broadened by the French academic engineers to measure any work done by any motor engine, and by any other acting agent (man, horse
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At the same time, although the term ‘work’ did not lose its empirical meaning, it underwent a process of abstraction. Instead of height and weight, ‘work’ was determined by the body’s displacement and the moving force (whatever this force might be) acting in any direction in relation to the displacement. Also, the conditions under which ‘work’ was produced were exactly determined (Kanderakis, 2010).

iv. Finally, this magnitude was connected to other magnitudes of mechanics (e.g. ‘living force’), that is it acquired systemic meaning (Baltas, 1988), and was organically incorporated in the theory of mechanics.

History of Science in the Science Classroom

History of science has been used in science education in many ways and for many purposes. Its first systematic introduction in secondary education appeared in the 60s and 70s, when history of science was used as a structural principle of the curriculum. An exemplary case of this approach was the ‘Harvard Project Physics’, which appeared in the USA in the early 70s (Rutherford, Holton & Watson, 1981). Although these programs improved students’ positive attitudes towards science, they failed to promote the content knowledge of science (Dedes, 2005; Koliopoulos, 2012). One of the causes for this failure was probably the multiplication of the information students had to handle, and the difficulty to discriminate today’s concepts from the old ones. For example, in addition to Newtonian mechanics, students had also to understand the complexities of the Aristotelian physics, and navigate between them, which was not easy task.

From the 80s until now, science educators have suggested a more local use of the history of science (Dedes, 2005). The latter was proposed as a means of teaching and learning the content and the methods of science (Seroglou & Koumaras, 2001; Stinner et al., 2003; Dossis & Koliopoulos, 2005; Dedes & Ravanis, 2009; Henke, Hötting & Riess, 2009), and also as an effective means of teaching the nature of science (Seroglou & Koumaras, 2001; Clough & Olson, 2004; Olson et al., 2005; Galili, 2008; Koliopoulos, 2012).

An important part of these publications and suggestions is not addressed to students but to teachers, pointing out several benefits the history of science can offer to them. Some of the advantages are:

- Teachers can gain some understanding of the nature of science and the relations between science and culture. Consequently, it is expected of them to change their classroom practices and implicitly convey some of the nature of science features to their students (Abd-el-Khalick, Bell & Lederman, 1998; Koliopoulos, 2012).
- Recognizing the difficulties scientists of the past had in creating and disseminating new concepts, teachers can acknowledge students’ cognitive obstacles to change their intuitive conceptions (Koliopoulos, 2012).
Teachers can use models and arguments, employed by scientists of the past to create, explain and disseminate new scientific concepts in order to help their students to master them (Galili, 2008).

The teaching approach proposed in this article uses exactly this last practice. First, it tries to introduce ‘work’ as a measure of work, in accordance with its history, and in a way which is meaningful for the student. Moreover, it uses some of the stages the creation of ‘work’ went through history to design students’ activities. Also, it employs historical arguments, the pioneers who created ‘work’ had used to convince their peers, in order to guide today’s students into a general formula for the calculation of ‘work’. Consequently, the historical material has an indirect influence on the teaching sequence: it formulates its main goal and to a degree the design of the students’ activities.

It has been pointed out that science teachers are usually reluctant to implement history and philosophy of science (HPS) material into science teaching. The majority of them do not have good knowledge of HPS, nor are they willing to make serious effort to learn about it. Besides, they do not know how to use HPS material to teach the content of science, which they regard as the most important objective of their job (Höttecke & Silva, 2011). Although it requires some knowledge of the history of ‘work’, this teaching approach is a mild intervention in the usual physics curriculum, and it is easier for teachers to accept. Moreover, it overcomes two frequent objections against HPS teaching proposals: the lack of instruction time (because of the extra historical material), and the exposure of students to erroneous past theories and unfamiliar past concepts, which could cause, as it is claimed, confusion in the students’ minds (Galili, 2008).

The Teaching Approach

Traditionally, ‘work’ is introduced by an equation \( W = F \cdot s \) or \( W = F \cdot s \cdot \cos \theta \), where \( \theta \) is the angle between the force vector and the displacement), usually without direct connection to the affairs of the world, that is without empirical meaning. Only later on, ‘work’ will be connected with other magnitudes of physics, such as kinetic energy or heat and internal energy, acquiring systemic meaning (Baltas, 1988; Kanderakis, 2014).

This teaching approach proposes something different. First, it introduces ‘work’ as a measure of work. Second, the proposed students’ activities are inspired by the details of the history of the magnitude ‘work’. They concern different working agents (humans, motor engines etc) in different circumstances, and they connect this magnitude to the everyday world, giving it (empirical) meaning. Third, the arguments that the creators of ‘work’ employed to persuade their colleagues are picked and recorded so that
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teachers can use them to handle difficulties in the classroom. Fourth, it follows, at least in part, the stages the evolution the magnitude of ‘work’ went through since its creation, using students’ experiences with manual work.

The teaching process is addressed to first grade upper Greek secondary school students (15-16 years old). The specific goals learners have to achieve, which serve the main goal, are:

a. To calculate ‘work’ in a vertical transport of a body, as a measure of a human’s work to raise it, with the product of the body’s weight and the height of its elevation.

b. To use a similar magnitude in order to measure the work done by a human in a horizontal transport of a body.

c. To calculate the work (that is ‘work’) produced by a motor engine in a horizontal transport of a body.

d. To determine the conditions under which ‘work’ is produced.

e. To come up with a general formula in order to calculate the ‘work’ produced by any acting agent at any circumstances.

In order to obtain these goals, a series of students’ activities are proposed. The activities are inspired by the stages the development of the magnitude ‘work’ went through its creation in history.

1. Initially, students are prompted to find a way to measure work through a simple problem based on an everyday situation. Learners are asked to compare the work of two men raising sacks of cement.

2. A series of hands-on activities where students raise bricks of different weights at different heights aims at showing that the amount of work (‘work’) is proportional to the height of elevation and the weight raised, that is to pursue the first goal. These are not only paper and pencil activities, but hands-on activities conducted with real bricks.

3. To work towards the second goal, and extend the use of ‘work’ in a horizontal transport of a body, students push and move horizontally heavy boxes (or tables), and discuss their estimation of the work done. In this case, however, students have to consider the horizontal moving force (overcoming resistant forces) and not the body’s weight, as Coulomb had done in the 18th century. The teacher can use Coriolis criticism on Coulomb to convince his students for this. Coriolis argued that, transporting a body horizontally, one can drag, with the same horizontal force, very different weights, depending on friction and other resistant forces. Hence, ‘work’ has to be calculated by the moving horizontal force (vanquishing resistances) multiplied by its displacement and not by the weight of the body moved multiplied by its displacement (see previous section).

4. Initially, the only acting agent was human. To pursue the third goal, a paper and pencil
activity is given to students: now the acting agent is a motor engine (a toy car) which drags a small but heavy box and students are asked to estimate its work.

5. To strive for the fourth goal, another series of hands-on activities are given to students to determine under what circumstances humans do work and produce ‘work’. The focus is on the work’s result and not on students’ fatigue, since ‘work’ was created to measure the produced result and nothing else. Teachers can employ here the relevant arguments of Poncelet. As we have seen (see the section on the history of ‘work’), Poncelet convincingly argued that when the effort or the acting force did not create any movement at all, one could easily replace the acting agent with an inert body (i.e. a pillar), which of course did not work (or produce ‘work’). Consequently, a body’s displacement is a necessary condition to produce ‘work’.

6. In order to achieve the fifth goal students discuss and generalize their findings and are guided to put forward a single general formula with the help of which one can calculate the ‘work’ produced by any agent at any circumstances.

7. Under the light of this new knowledge students are asked to evaluate their efforts on the introductory problem and seek new answers.

8. Finally, a metacognitive discussion is carried out focusing on the practices students employed to come to this knowledge.

The teaching process requires two teaching sessions (90 minutes). For more details see the student’s worksheet in the Appendix.

**Discussion**

The aim of the teaching approach described in the article is to encourage active learning where pupils interact with the material by doing things, answering questions, completing simple calculations, drawing conclusions, and finally gaining understanding. All the above are put at the service of the main goal of the proposed teaching process, which is to teach ‘work’ as a measure of work, in accordance with the history of its creation. Another idea of the teaching approach is to use some of the stages engineers of the past went through to create ‘work’ as guidelines for designing students’ activities.

Teachers need to acquire some knowledge of the history of ‘work’ in order to embrace the main goal (to teach ‘work’ as a measure of work) as well as to understand the significance of the different stages ‘work’ went through when the engineers constructed it in the first place. They also need to be familiar with the reasoning these engineers employed in order to frame the definition of the magnitude ‘work’ and the conditions for its use.

- The first activity (the introductory paper and pencil problem) just poses the issue
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of the calculation of work in a familiar situation. Usually students do not have the resources to deal successfully with it at that moment.

• Next, with a group of hands-on activities, the measure of work (that is the magnitude of ‘work’) is introduced, using the example of a vertical transport of a rigid body by a human. The idea is derived from the British engineers who created the quantity ‘weight multiplied by height’ to measure the work of steam engines raising water. These activities are related to students’ everyday experiences and can be followed without much difficulty.

• In the following activity, students extend the concept of ‘work’ (using the same magnitude) to measure the work carried out by a human in a horizontal transport of a body. Although the situation is slightly different, this activity is in some ways similar but not the same to what the 18th century British engineers did. The engineers used the measure ‘weight multiplied by height’ to estimate their steam engines’ effectiveness for their new tasks (moving other engines). However, today’s students should be persuaded not to apply blindly this measure, multiplying displacement by the transported body’s weight, but use the horizontal moving force instead (equal to the resistance in smooth movements). Coriolis’ arguments (mentioned in the previous sections) are usually enough to convince students to use this force for the calculation of ‘work’ (and work).

• In the next activity, the magnitude of ‘work’ is extended to measure the work of a motor engine that transports a body horizontally. Although it is a paper and pencil activity, a real toy car dragging a matchbox full of nails can make the situation more concrete. The calculation of a motor engine’s work was the first process we encountered in the history of ‘work’ even though in a different task: to raise water. Today’s students having coped well with the previous activity can easily replace the acting human agent with a motor engine.

• During the following activity students are asked to determine the conditions under which work is carried out and ‘work’ is produced. This was a late development in the evolution of the magnitude of ‘work’ and concerned the elaboration of the concept of ‘work’ and the determination of its field of application. It is probably the hardest notion students have to come to terms with and needs time and discussion to carry out. The arguments of Poncelet (mentioned in the previous sections) are helpful though.

• The outcomes we expect from the metacognitive discussion (how we have found what we found?) are: hands-on activities (experiences rather than experiments) or experiences in general and reasoning. Since students will probably have difficulties arriving to general and clear outcomes, teacher’s discreet interference is required.

This teaching process is constructed to be in accordance with the existing physics curriculum. However, other teaching approaches could be designed, where the historical
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phenomenological field itself (the field of phenomena and artifacts relevant to the history of ‘work’ - that is water wheels, steam engines etc) could be used to guide the formulation of teaching activities.

The proposed teaching process intends only to introduce ‘work’. To comprehend ‘work’ as a magnitude of physics, one has also to connect it with other physical magnitudes (i.e. ‘kinetic energy’), giving it systemic meaning (Baltas, 1988). Good tools for this are the 18\textsuperscript{th} century experiments of Poleni (Maffioli, 1994) and Gravesande (1774), who let small hard balls fall upon soft material and measured the falling balls’ ‘force’ by the size of the produced cavities. The experiments were designed to show that this ‘force’ was proportional to the square of the body’s velocity (and consequently to its vis viva), but they can be used today to show that the ‘work’ done by the force of gravity (weight times height) is proportional to the ‘kinetic energy’ a falling body gains, estimated by the cavities’ size (Kanderakis, 2014).

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\textsuperscript{3} To be exact, the experiments showed that the falling body’s ‘force’ was proportional to the height of fall. But it was known (from Galileo) that this height was proportional to the square of the body’s final velocity.
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APPENDIX

Student’s worksheet

THE MAGNITUDE ‘WORK’ MEASURES WORK

i. Introductory problem

Sakis and Manos raise 25kg sacks of cement to an apartment building. In an hour, Sakis raises 15 sacks to the 2nd floor, while Manos raises 12 sacks to the 3rd floor.

Who will work more?

Each floor and the basement have 3.5m height. The 25kg sack has 250N weight.

Image 1: Introductory problem

ii. Calculating ‘work’

Activity 1

Kostas raises a brick (with 1kg mass and 10N weight) from the floor to the table in a height of 1m.

Vaso raises the same brick to a height of 2m.

Question

Whose work is greater?

Explain your answer.

Image 2: Calculating ‘work’ 1
For a vertical transport of a body with a weight \( w \) to a height \( h \), the measure of work is proportional to:

1. 
2. 

The measure of work can be found by multiplying these two magnitudes. So,

\[
\text{Measure of work} = \frac{w}{1} \times \frac{h}{1}
\]

This measure of work is called ‘work’, and its symbol is \( W \).

Using symbols, the above equation becomes \( W = \ldots \).

### iii. Extending the new knowledge in a horizontal transport of a body

#### Activity 3

Maria pushes a 5kg box (with 50N weight) and moves it 3m very slowly. The force of friction, which resists motion, is 14N.

With what horizontal force has Maria to push the box in order to put it in motion?

What is the magnitude of ‘work’ that Maria has produced?

**Image 4: Extending ‘work’**
In a horizontal transport of a body:

‘Work’ (measure of work) = \[ \text{force} \times \text{distance} \]

Using symbols, \( W = \text{force} \times \text{distance} \)

**iv. Calculating the work of motor engines**

**Activity 4**

A battery-driven toy car drags horizontally a full of nails small box. The toy car moves slowly just overcoming friction, and applies horizontal force 0.8N to the box. The box is carried 0.4m forward.

What is the magnitude of ‘work’ that the toy car produces?

*Image 5: Extending ‘work’ 2*

When a motor engine transports a body horizontally, the ‘work’ it produces (and the work done) is:

‘Work’ = \[ \text{force} \times \text{distance} \]

In symbols, \( W = \text{force} \times \text{distance} \)

**v. When is ‘work’ produced?**

**Activities 5**

a. Push the wall with your hands.
b. Keep your schoolbag for five seconds.
c. Push the table to move it two meters.
d. Raise your schoolbag from the floor to the table.

**Questions**

When do you produce ‘work’?
When you don’t produce ‘work’?
Explained your answers.

*Image 6: When ‘work’ is produced?*
‘Work’ is produced (and work is done), when two things happen at the same time:
A ---------------- acts on a ----------------.
The ---------------- puts the body in ----------------.

vi. Generalization
In the general case of the transport of a body by applying a force (F) in the direction of the body’s displacement (s):
‘Work’ = -----------------multiplied by ----------------------.
In symbols, W = -------------.

vii. Solving the introductory problem
Go back to the introductory problem. Using what we have found, make your calculations, and find the answer.

viii. How do we found this formula?
According to your opinion, what were the most important things we did that leaded us to the ‘work’ formula.